Δευτέρα, 9 Οκτωβρίου 2

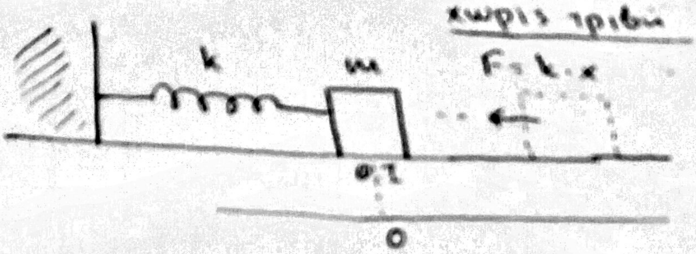
Διαφορικές Εξισώσεις (Πουρναρας)

$v'(t) = t^2$, $t\in\mathbb{R}$

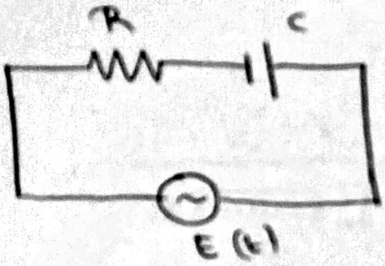
Ψάχνουμε την $v(t)$.

$$\int_a^t v'(s)\,ds = \int_a^t s^2\,ds = v(t) - v(a) = \left[\frac{s^3}{3}\right]_a^t$$

ΕΦΑΡΜΟΓΕΣ



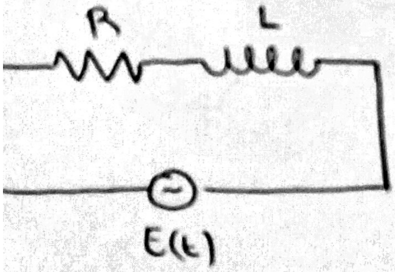
$$k\cdot x = m\cdot g = m\cdot x''$$



$$E_R = R\,I(t) \qquad E_C = \frac{Q(t)}{C}$$

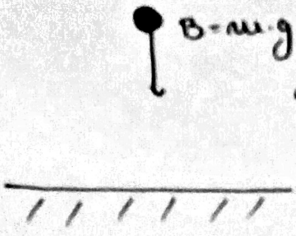
$$E(t) = E_R + E_C \Rightarrow E(t) = R\cdot I(t) + \frac{Q(t)}{C} \Rightarrow$$

$$E(t) = R\,\frac{dQ(t)}{dt} + \frac{1}{C}\,Q(t) \Rightarrow R\cdot Q'(t) + \frac{1}{C}Q(t) = E(t)$$



$$E_L = L\cdot I'(t) \qquad E_R = R\cdot I(t)$$

$$E(t) = E_L + E_R = L\,I'(t) + R\cdot I(t)$$



Mo

$B = m\cdot g = m\cdot x'' = mg$

$m\cdot g - \underbrace{k\cdot y}_{\text{αντίσταση αέρα}} = m\,x''$

Πληθυσμιακά Μοντέλα (Population Models)

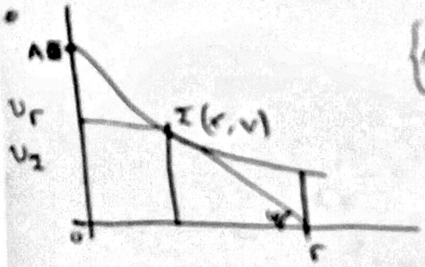
Έστω ότι μετράμε τον πληθυσμό $x(n+1) - x(n) = a x(n) - b x(n) = k x(n) \Rightarrow$

επόμενη χρονική μονάδα — χρονική μονάδα

$\Rightarrow x(n+1) - x(n) = k x(n)$

$x(t+\Delta t) - x(t) = k x(t) \Delta(t) \Rightarrow \frac{x(t+\Delta t) - x(t)}{\Delta(t) \to 0} = k x(t) \Rightarrow x'(t) = k x(t)$

$x'(t) = (a - b x(t)) x(t)$ Λογιστική Εξίσωση



{Πότε ο σκύλος πιάνει τη γάτα;}

$O\Gamma = u_\Gamma \cdot t = OB + B\Gamma = x - \frac{y}{y'}$

$A\Sigma = u_\Sigma t = \underbrace{\int_0^x \sqrt{1+(y')^2}\, ds}_{\text{μήκος}}$

$\tan\varphi = \frac{B\Sigma}{B\Gamma} \Rightarrow B\Gamma = \frac{y}{\tan\varphi} = \frac{y}{y'}$

$\frac{u_\Gamma}{u_\Sigma} = \frac{x - \frac{y}{y'}}{\int_0^x \sqrt{1+(y')^2}\, ds} \Rightarrow$

$\Rightarrow \left(x - \frac{y}{y'}\right) \frac{u_\Sigma}{u_\Gamma} = \int_0^x \sqrt{1+(y')^2}\, ds \Rightarrow \frac{u_\Sigma}{u_\Gamma}\left[\quad\right]' = \sqrt{1+(y')^2}\, ds$

Ιστορική Αναδρομή

- 1671 Newton → οι πρώτες διαφορικές εξισώσεις, πχ: $y' = 1 - 3x + y + x^2 + xy$

3 κατηγορίες

- $y'(x) = f(x)$
- $y'(x) = f(x, y)$
- $\frac{\partial x}{\partial y}, \frac{\partial y}{\partial z}$

- 1676 Leibnitz: $\frac{dy}{dx} = y'$, γεωμετρικά προβλ. εφαπτ.